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A Brane model with two asymptotic regions.

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Some brane models rely on a generalization of the Melvin magnetic universe including a complex scalar field among the sources. We argue that the geometric interpretation of Kip.S.Thorne of this geometry restricts the kind of potential a complex scalar field can display to keep the same asymptotic behavior. While a finite energy is not obtained for a Mexican hat potential in this interpretation, this is the case for a potential displaying a broken phase and an unbroken one. We use for technical simplicity and illustrative purposes an ad hoc potential which however shares some features with those obtained in some supergravity models. We construct a sixth dimensional cylindrically symmetric solution which has two asymptotic regions: the Melvin-like metric on one side and a flat space displaying a conical singularity on the other. The causal structure of the configuration is discussed. Unfortunately, gravity is not localized on the brane.

PACS numbers:

I. INTRODUCTION

Among the most important characteristics of cosmic strings is the existence of a symmetry axis and the concentration of energy around this axis [1]. Taking gravity into account, the existence of a symmetry axis implies cylindrical symmetry for the metric as well. The static cylindrically symmetric solutions of Einstein equations in vacuum in $4D$ are of the Kasner type: they are parameterized by three constants obeying two constraints. The vanishing of the energy-momentum tensor in the asymptotic region implies that the geometry must approach a Kasner line there. As the energy momentum tensor corresponding to these axial configurations implies the invariance of the metric under boosts, one is left with only two Kasner geometries : a flat space presenting a conical singularity and a Melvin like space [2]. The first case leads to the well known cosmic strings. The second solution, written in a particular system of coordinates, displays circles of decreasing circumferences for increasing "radii"ρ. This feature has been analyzed by Kip.S.Thorne [3, 4]. The interpretation is that $\rho = \infty$ is the point at infinity on the symmetry axis.

The Melvin solution has high dimensional generalizations which can be used to build brane models [5, 6]. In recent works, complex scalar fields have been incorporated into the picture [7]. In this article we address the same question. However, we use the Kip S.Thorne interpretation to fix the boundary conditions; the coordinate ρ being the point at infinity on the symmetry axis, the angular coordinate is not well defined there, just as for the polar coordinates at the origin on the plane. A cylindrically symmetric complex field must thus vanish at that point. We obtain that no static cylindrical solution can be obtained with the usual Higgs potential. We then exhibit a toy model for which this can be achieved and discuss its characteristics.

This paper is organized as follows. In the second section we review the geometric interpretation of The Melvin magnetic solution in four dimensions. The third section incorporates the scalar field, set the boundary conditions and displays the numerical solution obtained. First, an Abelian-Higgs Lagrangian is coupled to the Einstein-Hilbert one. Looking for an axially symmetric configuration which displays the second special Kasner line element far from the source , the preceding section imposes the vanishing of the vector and the scalar field as the coordinate ρ goes to infinity. This results in a divergent inert mass per unit length if the v.e.v of the Higgs field does not vanish. If it does, one simply recovers the Melvin solution. We then construct, for illustrative purposes, a potential for which the behavior of the scalar is non trivial. This potential has two minima, one of them being at zero. This has similarities with some potentials obtained in some supergravity [7] models and some non commutative models [8]. The constructed solution interpolates between them. The minimum corresponding to a vanishing value of the field is attained in the Kasner-like asymptotic region while the non vanishing value corresponds to a flat space presenting an angular deficit. The classical trajectories of neutral particles in this geometry are analyzed in section four. We show

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that the trajectories of massive particles in this geometry are bounded. We also study massless particle trajectories and discuss the causal structure of the solution. An appendix is devoted to the way the numerical approximation has been computed.

The coupling of scalar fields to gravity leads to many classical solutions [9, 10, 11, 12, 13, 14, 15, 16]. The introduction of a scalar field among the sources leading to a Melvin type universe has been considered before [17]. There are three main differences between our work and the previous papers. Firstly, the scalar field considered here is not a dilaton so that its coupling to gravity is ordinary. The involved potentials are not the same. Secondly, the scalar field here is complex, contrary to [17] where it is real. The vanishing of the scalar field on the symmetry axis is not required when it is real. On the contrary, this becomes mandatory when it is complex, just like for the Higgs field on the cosmic string core. But, the more important difference is that here we have a configuration with two asymptotic regions. Like in [37], we do not have a delta function for the brane.

II. THE MELVIN SOLUTION IN $4D$.

In this section we review the geometric interpretation of the Melvin solution. To keep the discussion as simple as possible, we actually analyze its asymptotic limit in four dimensions. The conclusions are however the same in the presence of extra dimensions and the addition of matter.

We will study a system consisting of a self gravitating Maxwell system in d dimensions. The classical field equations are derived from the action

$$S = \int d^d x \sqrt{|g|} \left[-\frac{1}{4} F_{ab} F^{ab} + \frac{R}{16\pi G} \right] . \tag{1}$$

The solution which generalizes the Melvin universe in d dimensions is given by the following expressions of the metric and the Maxwell tensor:

$$\begin{aligned} ds^2 &= \left(1 + \frac{\rho^2}{a^2}\right)^{2/(d-3)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - d\rho^2\right) - \rho^2 \left(1 + \frac{\rho^2}{a^2}\right)^{-2} d\phi^2 \quad , \\ F &= B_0 \left(1 + \frac{\rho^2}{a^2}\right)^{-2} \rho \, d\rho \wedge d\phi \quad , \end{aligned} \tag{2}$$

a and B_0 being dependent constants. This solution has been used in the brane world scenarios. One of its important properties is that the brane can have positive tension and the closure of the bulk provides a singularity-free boundary condition for solutions that contain black holes and gravitational waves on the brane [5, 6].

The characteristic of this metric on which we will put the emphasis is the fact that the circumference of circles obtained by letting only ϕ non fixed tend to zero as the coordinate ρ goes to infinity. The Lagrangian from which the solution given in Eq.(2) follows did not contain any scalar field. The question we address in this paper is: if one adds a scalar field to the picture, which kind of potential allows the same behavior for the metric? We will argue that a geometric interpretation of this property of the metric gives an important restriction.

The features observed by Kip.S.Thorne for the Melvin solution [18, 19] are already present in a vacuum solution: the Kasner line element [2] which is obtained by taking $a = 0$ in Eq.(2). The simplest example where this behavior can be analyzed is the sphere. Introducing the polar stereographic coordinates (r, θ), the metric of a sphere of radius σ reads [20]

$$ds^2 = \frac{16\sigma^4}{(4\sigma^2 + r^2)^2} dr^2 + \frac{16\sigma^4 r^2}{(4\sigma^2 + r^2)^2} d\theta^2 \quad . \tag{3}$$

For large values of the coordinate r, the coefficient $g_{\theta\theta}$ becomes a decreasing function . If one interprets r as the radius, a circle of infinite radius turns out to be of null length. This is obvious since $r = \infty$ corresponds to the point at infinity on the plane which is mapped into the north pole by the stereographic projection; $r = \infty$ is just the north pole. When the coordinate r vanishes, one has another circle displaying a vanishing circumference : the south pole. The two are on the symmetry axis. Introducing the variable

$$r_* = 2\sigma \arctan(r/2\sigma) \quad , \quad \text{the metric reads} \quad ds^2 = dr_*^2 + \sigma^2 \sin^2\left(\frac{r_*}{\sigma}\right) d\theta^2 \quad . \tag{4}$$

The relation between r_* and r is bijective provided that $r_* \in [0, \pi\sigma]$. The points located on the symmetry axis once again are those for which the coefficient $g_{\theta\theta}$ vanishes.

III. THE $6D$ EXTENSION WITH A COMPLEX SCALAR FIELD.

Before proceeding, let us point out that the tangential Maxwell vector field in the Melvin solution vanishes when ρ goes to infinity, in accord with the geometric interpretation. We now wish to include a scalar field in the picture, in the presence of extra dimensions. We choose a sixth dimensional model essentially because in this case one can naturally obtain chiral fermions.

Let us first consider a scalar displaying a Higgs potential; the matter action is then

$$S = \int d^6x\sqrt{-g}\left[\frac{1}{2}D_\mu\Phi D^\mu\Phi^* - \frac{\lambda}{4}(\Phi\Phi^* - v^2)^2\right]. \tag{5}$$

The $U(1)$ charge e is embodied in the covariant derivative $D_\mu\Phi = \partial_\mu\Phi - ieA_\mu\Phi$. For a static cylindrically symmetric configuration, the ansatz can be given the form

$$\begin{aligned} ds^2 &= \beta^2(\rho)\eta_{\mu\nu}dx^\mu dx^\nu - \gamma^2(\rho)d\rho^2 - \alpha^2(\rho)d\phi^2 \quad \text{where} \quad \mu = 0,\cdots 3 \quad , \\ \Phi &= vf(\rho)e^{i\phi} \quad \text{and} \quad A_\phi = \frac{1}{e}(1-p(\rho)) \quad . \end{aligned} \tag{6}$$

The cosmic string solution has been extensively studied in the literature. In that configuration, the smoothness of the geometry on the symmetry axis is guaranteed by going to the gauge $\gamma(\rho) = 1$ and imposing the boundary conditions [2]

$$\alpha(0) = 0, \alpha'(0) = 1 \quad , \tag{7}$$

while the matter fields are non singular on the core provided that

$$f(0) = 0\ ,\ p(0) = 1 \quad . \tag{8}$$

The vanishing of the energy density in the asymptotic region implies

$$f(\infty) = 1 \quad , \quad p(\infty) = 0 \quad . \tag{9}$$

What happens if we want the metric to display the same asymptotic behavior than in Eq.(2) when the coordinate ρ goes to infinity? In the previous section, we argued that $\rho = \infty$ is the point at infinity on the symmetry axis. To have a regular cylindrically symmetric configuration, the Higgs and the tangential vector field must vanish there:

$$f(\infty) = 0 \quad \text{and} \quad p(\infty) = 1 \quad . \tag{10}$$

Extracting the expression of the integrand of the inertial mass from Eq.(5) one has

$$\epsilon(\rho) = \sqrt{|g|}\left[\frac{1}{2}g^{\rho\rho}|D_\rho\Phi|^2 + \frac{1}{2}g^{\phi\phi}|D_\phi\Phi|^2 + \frac{1}{4}F_{\rho\phi}F^{\rho\phi} + \frac{\lambda}{4}(\Phi^*\Phi - v^2)^2\right]. \tag{11}$$

In the asymptotic region (i.e. $\rho \to \infty$) one has $\sqrt{-g} \sim \rho^{7/3}$; the volume element is not bounded. The first three terms decrease in the asymptotic region provided that $f(\rho)$ and $p(\rho)$ approach constants there; this is already satisfied by Eq.(10). The contribution of the Higgs potential in this part of the space is reduced to the integral of $\rho^{7/3}\lambda v^4$. This has a chance to converge only when $v = 0$. Then, the parameterization given in Eq.(6) does not apply; one can however define a dimensionless function associated to the Higgs field by using the Newton constant. Doing this, we obtained a vanishing scalar field for any value of the parameters.

Physically this can be understood as follows. Forcing the scalar field to go from zero to zero as ρ goes from zero to infinity, one obtains that it vanishes identically since there is no source. Such a source would be for example a local maximum of the potential but as the vacuum expectation value vanishes, such a maximum does not exist. In fact, one recovers the Melvin universe.

Is it possible to construct a solution with a non vanishing scalar field? To do this we need a potential which vanishes with the scalar field so that the minimum is attained at spatial infinity. We also need a local maximum which will correspond to a source. These conditions are for example satisfied by the gauge invariant potential

$$V(\Phi) = \lambda e^{w^2\Phi\Phi^*}\Phi\Phi^*(\Phi\Phi^* - v^2)^2 \quad . \tag{12}$$

The maximum is attained at $\phi = \pm v\sqrt{\sqrt{2}-1}$ while there are three minima , at $\phi = 0, \pm v$. The U(1) symmetry is broken spontaneously in the last two vacua and preserved in the first one. We disregard the renormalizability since

we are interested only in classical solutions. This potential, although purely ad hoc, shares with the one appearing in [7]

$$V(\phi) = 2e^{\phi\bar{\phi}}(\phi\bar{\phi})^{p-1}\left[2(p+\phi\bar{\phi})^2 - 5\phi\bar{\phi}\right] \tag{13}$$

the fact that it is the product of an exponential and a polynomial. The difference is the fact that in Eq.(13), one needs to have $p \geq 1$ to have the vanishing value of the field as a vacuum but then there is no other vacuum. In [7], it was argued that the potential of Eq.(13) could be seen as inspired from some supergravity model, with a particular choice of the Kahler structure. Let us also mention that in models in which one works with non commutative spaces such as $\mathcal{M}\times M_n$ where $\mathcal{M}$ denotes the Minkowski space and M_n the set of $n\times n$ matrices, one also obtains potentials displaying symmetric vacua. The attitude adopted here is like the one concerning analytical solutions for self gravitating domain walls [22, 23, 24]. One knows that a potential which is a cosine of a scalar field achieves the desired goal, although it is not renormalizable. In the same way, one introduces ad hoc potentials for quintessence [25, 26, 27]. Some of them are negative powers of a scalar field and so lead to non renormalizable theories. Our only aim is to show that solutions with finite energy exist for specific potentials. Moreover, some potentials displaying symmetric vacua have been used as candidates for dark matter [28, 29].

We now wish to construct a solution which interpolates between two vacua, say $|\Phi| = v$ and $|\Phi| = 0$. Our previous discussion tells us that the region where the field goes to its unbroken phase can not be Melvin-like. As we simply require cylindrical symmetry, we can choose that asymptotic region to be like the far region of a cosmic string.

The Einstein equations will be written in the form

$$R_a^b = -8\pi G\left(T_a^b - \frac{1}{4}T\right) \tag{14}$$

where the factor 1/4 comes from the dimension of the space time. In sixth dimensions, asking our action to be a pure number means the self coupling λ and the gauge coupling e are dimensionfull. We thus can write our equations in terms of the dimensionless parameters

$$\mu = Gv^2 \quad , \quad \nu = \lambda^2 G^{-3} \quad , \quad \tau = e^2 v \quad \text{and} \quad \sigma = w^2 v^2 \quad . \tag{15}$$

Note that here the dimension of v is an inverse length square. We will use the dimensionless length x given by

$$\rho = L\,x \qquad \text{where} \quad L = \frac{1}{2\sqrt{\pi}\mu^{5/4}\nu^{1/4}}\frac{1}{\sqrt{v}} \tag{16}$$

and the dimensionless functions

$$\alpha(\rho) = \sqrt{\pi}\sqrt{\frac{\mu}{\tau}}\frac{1}{\sqrt{v}}A(x) \quad , \quad \beta(\rho) = B(x) \quad , \quad \gamma(\rho) = \bar{\gamma}(x) \quad , \quad f(\rho) = F(x) \quad , \quad p(\rho) = P(x) \quad . \tag{17}$$

The field equations read

$$e^{\sigma F^2(x)}B(x)F^2(x)\bar{\gamma}^2(x)(1-F^2(x))^2 + \frac{A'(x)B'(x)}{A(x)} + 3\frac{B'^2(x)}{B(x)} - \frac{B'(x)\bar{\gamma}'(x)}{\bar{\gamma}(x)} - 2\frac{B(x)P'(x)^2}{A^2(x)} + B''(x) = 0 \tag{18}$$

$$\frac{1}{4}e^{\sigma F^2(x)}B(x)F^2(x)\bar{\gamma}^2(x)(1-F^2(x))^2 + 2\pi\mu B(x)F'^2(x) - \frac{B(x)A'(x)\bar{\gamma}'(x)}{4A(x)\bar{\gamma}(x)} - \frac{B'(x)\bar{\gamma}'(x)}{\bar{\gamma}(x)} + 3\frac{B(x)P'^2(x)}{2A^2(x)} + \frac{B(x)A''(x)}{4A(x)} + B''(x) = 0 \quad , \tag{19}$$

$$e^{\sigma F^2(x)}B(x)F^2(x)\bar{\gamma}^2(x)(1-F^2(x))^2 + \frac{2\tau}{\pi\mu^{5/2}\sqrt{\nu}}\frac{F^2(x)\bar{\gamma}^2(x)P^2(x)}{A(x)} + 4\frac{A'(x)B'(x)}{B(x)} - \frac{A'(x)\bar{\gamma}'(x)}{\bar{\gamma}(x)} + 6\frac{P'^2(x)}{A(x)} + A''(x) = 0 \quad , \tag{20}$$

$$\frac{\tau}{2\pi\mu^{5/2}\sqrt{\nu}}F^2(x)\bar{\gamma}^2(x)P(x) - \frac{A'(x)P'(x)}{A(x)} + 4\frac{B'(x)P'(x)}{B(x)} - \frac{\bar{\gamma}'(x)P'(x)}{\bar{\gamma}(x)} + P''(x) = 0 \quad , \tag{21}$$

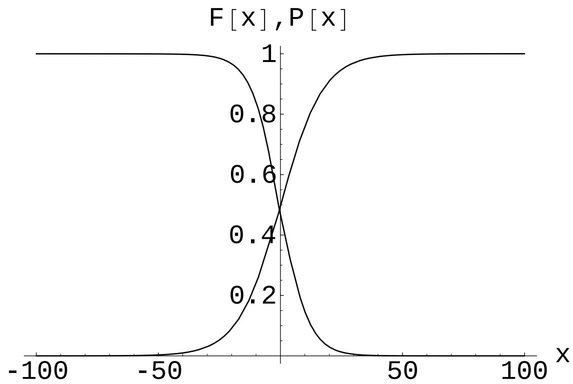

FIG. 1: The scalar and vector fields are plotted in terms of the coordinate x.

$$\frac{1}{2\pi\mu}e^{\sigma F^2(x)}F(x)\bar{\gamma}^2(x)(1+(-4+\sigma)F^2(x)+(3-2\sigma)F^4(x)+\sigma F^6(x))-\frac{\tau}{4\pi^2\mu^{7/2}\sqrt{\nu}}\frac{F(x)\bar{\gamma}^2(x)P(x)}{A^2(x)}$$
$$\frac{A'(x)F'(x)}{A(x)}+4\frac{B'(x)F'(x)}{B(x)}-\frac{F'(x)\bar{\gamma}'(x)}{\bar{\gamma}(x)}+F''(x)=0 \quad . \tag{22}$$

We now relax the assumption that the coordinate ρ goes from zero to infinity but rather take it to go from minus to plus infinity. This is like parameterizing every point of the sphere not by θ, ϕ but by ϕ and its distance from the equator on a meridian. The upper hemisphere would have positive ρ while the lower would correspond to negative values of that length coordinate. In our case, the region where $\rho \to -\infty$ will correspond to a cosmic string-like geometry while $\rho \to \infty$ will be associated with the Melvin-like behavior.

So, the boundary conditions are that when $\rho \to -\infty$, one is in the far region of the cosmic string solution:

$$A(x) \sim x \quad , \quad B(x) \sim 1 \quad , \quad \bar{\gamma} \sim 1 \tag{23}$$

while for $\rho \to \infty$ one enters the asymptotic region of the Melvin solution:

$$A(x) \sim \frac{a^2}{x} \quad , \quad B(x) \sim \bar{\gamma}(x) \sim \left(\frac{x^2}{a^2}\right)^{1/3} \quad . \tag{24}$$

To give an illustration, we need to solve the above set of coupled non linear differential equations. This has been done for the simplest choice of the parameters: $\mu = \nu = \tau = \sigma = 1$; the details concerning the numerical treatment are given in the appendix. The behavior of the different fields has been given in the pictures FIG.1, FiG.2, and FiG.3. Roughly speaking, the region $\rho = 0$ is where the transition between the two regions takes place; it is also the place where the energy is concentrated.

The trapping of gravity is ensured when the condition

$$\int dx^4 dx^5 g^{00}\sqrt{|g|} < \infty \tag{25}$$

is satisfied. From the asymptotic behavior of the metric, one sees this is not true for the solution we constructed. Let also remark that a change of coordinate can now be made so that for example $\bar{\gamma} = 1$.

IV. THE CLASSICAL TRAJECTORIES.

Let us first consider the case of massive particles. The metric specified in Eq.(6) has cyclic coordinates ; its geodesics are by way of consequence characterized by constants of motion. Introducing the proper time per unit mass τ, the energy-momentum relation reads

$$\left(\frac{d\rho}{d\tau}\right)^2 = \left(-1-\frac{k_1^2}{\alpha^2(\rho)}+\frac{k_2}{\beta^2(\rho)}\right)\frac{1}{\gamma^2(\rho)} \quad . \tag{26}$$

A physical motion is characterized by a real velocity. The asymptotic behavior of the metric displayed in Eq.(23,24) shows that the trajectory of a massive particle never attains the point at infinity on the Melvin branch; however it has access to the region at infinity on the string branch.

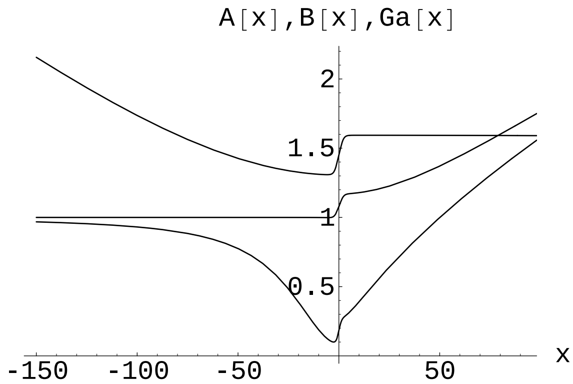

FIG. 2: From top to bottom, the components $A(x), B(x), \bar{\gamma}(x)$ of the metric

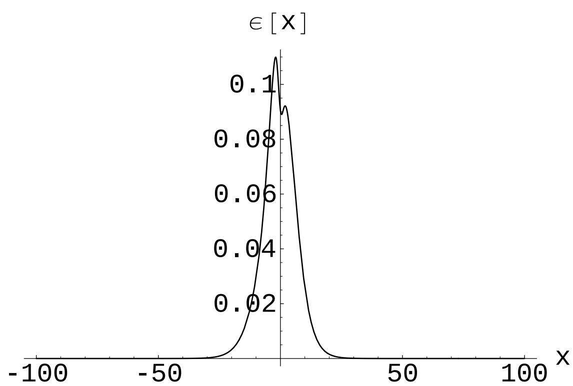

FIG. 3: The energy density $\epsilon(x)$

The causal structure is found by analyzing particular null geodesics. The bounded null coordinates in the new background are given by $\bar{u} = c^{st}$ or $\bar{v} = c^{st}$ with

$$\left.\begin{array}{c}\bar{u}\\ \bar{v}\end{array}\right\} = \arctan[t \mp \sigma(\rho)] \quad , \tag{27}$$

where

$$\sigma(\rho) = \int_0^\rho d\xi \frac{\gamma(\xi)}{\beta(\xi)} \quad . \tag{28}$$

In FIG.4, we have drawn the Penrose-Carter diagram of the solution.

The embedding of the two dimensional metric containing only t and ρ in the Euclidean space can be realized by the surface of revolution

$$Z(r) = \int_0^r dy \sqrt{c^2(y) - 1} \quad , \quad c(y) = \gamma(\alpha^{-1}(y)) \frac{d}{dy}(\alpha^{-1}(y)) \quad . \tag{29}$$

The limiting behavior of the metric shows that in the region $\rho \to -\infty$,

$$Z(r) \sim c^{st} r \quad ; \tag{30}$$

no restriction is imposed on r and the surface is a cone. On the contrary, as $\rho \to \infty$,

$$Z(r) \sim \int_{r_\star}^r dy \left[\left(\frac{a}{y} \right)^{16/3} - 1 \right]^{1/2} \quad ; \tag{31}$$

there is a maximal circumference.

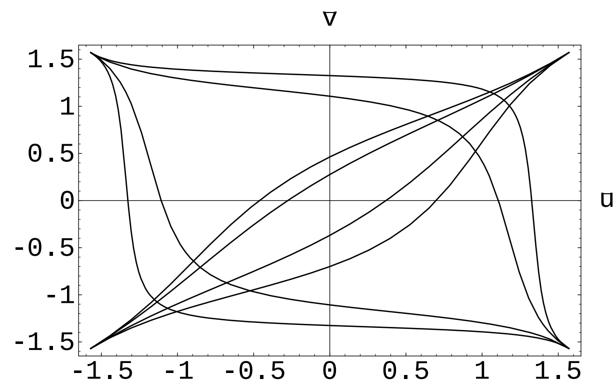

FIG. 4: The causal structure of the solution is given. The time-like infinities are $I^+(\bar{u} = \bar{v} = \pi/2)$ and $I^-(\bar{u} = \bar{v} = -\pi/2)$. As r can change sign, there are two space infinities : $I_o^>(-\pi/2, \pi/2)$ and $I_o^<(\pi/2, -\pi/2)$. The curves which begin at I^- and end at I^+ correspond to fixed values of r while the others correspond to fixed t.

V. CONCLUSIONS.

We have constructed a sixth dimensional cylindrically symmetric self gravitating configuration. It has two asymptotic regions: one corresponding to a flat space time with a deficit angle and the other to the second special Kasner line element. The part of the geometry which displays a deficit angle can be realized as a cone with a smoothed apex. The second special Kasner geometry, on the other side, can be seen as a tube with a decreasing radius. Gluing the two, one obtains something close to a funnel. The causal structure of this geometry was studied. It should be stressed that such a configuration, with two topologically different boundary regions, is not possible with static spherically symmetric configurations; the Birkhoff theorem forbids this.

The potential considered is non renormalizable but our discussion shows that it is one of the simplest which allows boundary conditions compatible with the geometric interpretation of the second special Kasner geometry.

Like in [32], we have built a configuration which has two different asymptotic regions. In our case we have a space with a conical singularity on one side and a Melvin-like solution on the other, while in the preceding one there are two Ads_5 space times glued together along a three brane. The fact that our solution is not asymptotically flat and the non localization of the four dimensional graviton is similar to [33]. Among the priorities which should be addresses if a more realistic model is built along these lines is the construction of realistic Abelian and non Abelian four dimensional models [34, 35].

What we have learned in this work is basically that if one wants to include a complex scalar field possessing a winding number on a Melvin-like solution, one needs a particular kind of potential. Our solution does not trap gravity. Nevertheless, the model may still have some phenomenological interest. Although we have not made here the appropriate analytic computations, one can not rule out at this stage the possibility of having a quasi localized four dimensional graviton on the brane like in [36]. In this model, it was shown that Newton's law of gravity was valid only between two length scales fixed by the theory.

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APPENDIX A: NUMERICAL CONSIDERATIONS.

Our numerical approximation relies on a symbolic approximation of the fields which can then be improved by a relaxation method. The function $F(x)$ goes from 1 to 0 when the argument goes from $-\infty$ to ∞. The ansatz is taken to be

$$F(x) = \frac{1}{2}(1 - \tanh{(f_0^2 x)})\frac{f_1^2 + f_2^2 x^2}{1 + f_2^2 x^2} \quad . \tag{A1}$$

In the same way, one has

$$P(x) = \frac{1}{2}(1 + \tanh{(p_0^2 x)})\frac{p_1^2 + p_2^2 x^2}{1 + p_2^2 x^2} \quad . \tag{A2}$$

The asymptotic behavior of the metric displayed in Eqs.(23,24) is taken onto account by the following functions:

$$\begin{aligned} A(x) &= \frac{1}{2}(1 - \tanh{(a_0^2 x)})(a_1^2 + a_2^2 x^2)^{1/2} + \frac{1}{2}(1 + \tanh{(a_3^2 x)})(a_4^2 + a_5^2 x^2)^{-1/2} \quad , \\ B(x) &= \frac{1}{2}(1 - \tanh{(b_0^2 x)})\frac{b_1^2 + b_2^2 x^2}{1 + b_2^2 x^2} + \frac{1}{2}(1 + \tanh{(b_3^2 x)})(b_4^2 + b_5^2 x^2)^{1/3} \quad , \\ \bar{\gamma}(x) &= \frac{1}{2}(1 - \tanh{(g_0^2 x)})\frac{g_1^2 + g_2^2 x^2}{1 + g_2^2 x^2} + \frac{1}{2}(1 + \tanh{(g_3^2 x)})(g_4^2 + g_5^2 x^2)^{1/3} \quad . \end{aligned} \tag{A3}$$

These parameters are not all independent, due to the fact one constant(a) drives the asymptotic behavior of the functions $\alpha(\rho), \beta(\rho), \gamma(\rho)$ and $P(\rho)$ simultaneously(see Eq.(2)). This has been taken into account. For simplicity, we introduce $\alpha_5 = \sqrt{a_5}$.

For the true solution, all the right members of the equations given in Eq.(18), $\cdot$ Eq.(22) , which we denote $ODE_1(x), \cdots, ODE_5(x)$, vanish. To obtain an initial approximation for the relaxation method, the idea is to look for the values of the coefficients $f_0, \cdots, g_5$ for which the integral

$$eq(x) = \int_{-\infty}^{\infty} dx \left(ODE_1^2(x) + ODE_2^2(x) + ODE_3^2(x) + ODE_4^2(x) + ODE_5^2(x) \right) \tag{A4}$$

is minimal.

Rather than computing the integral, we approximated the surface to which it corresponds by a sum of rectangles. The values of the parameters we found are given below. Plotting the functions $ODE_k(x)$, one finds an error of the order 10^{-2} *on the entire real axis*.

$$
\begin{aligned}
f_0 &= 0.29525844042277477\,,\, f_1 = 0.9658023897961495\,,\, f_2 = 0.21489954200347053\,,\, p_0 = 0.24041269454525904\,,\\
p_1 &= 0.9931697892207005\,,\, p_2 = 0.4970289653744458\,,\, a_0 = 0.8317552649976594\,,\, a_1 = 1.3064942111721376\,,\\
a_2 &= -0.011436440955002887\,,\, a_3 = 0.8012738041970792\,,\, a_4 = 0.6275485649917462\,,\, \alpha_5 = 0.019766866475879406\,,\\
b_0 &= 0.823008493904919\,,\, b_1 = 0.9933291170842984\,,\, b_2 = 0.42807374544606963\,,\, b_3 = 0.7966722551728198\,,\\
b_4 &= 1.2646707186251818\,,\, g_0 = 0.7842562408108704\,,\, g_1 = -0.2977638255573934\,,\, g_2 = -0.0349623024909614\,,\\
g_3 &= 0.8326686788126545\,,\, g_4 = 0.14292786831126408 \quad .
\end{aligned}
\tag{A5}
$$